Quine, W.V.O.

IN Three Ways of Paradox and Other Essays

15

# Three Grades of Modal Involvement

There are several closely interrelated operators, called *modal* operators, which are characteristic of modal logic. There are the operators of *necessity, possibility, impossibility, non-necessity.* Also there are the binary operators, or connectives, of *strict implication* and *strict equivalence.* These various operators are easily definable in terms of one another. Thus impossibility is necessity of the negation; possibility and non-necessity are the negations of impossibility and necessity; and strict implication and strict equivalence are necessity of the material conditional and biconditional. In a philosophical examination of modal logic we may therefore conveniently limit ourselves for the most part to a single modal operator, that of *necessity.* Whatever may be said about necessity may be said also, with easy and obvious adjustments, about the other modes.

There are three different degrees to which we may allow our logic, or semantics, to embrace the idea of necessity. The first or least degree of acceptance is this: necessity is expressed by a *semantical predicate* attributable to statements as notational forms—hence attachable to names of statements. We write, e.g.:

From the *Proceedings of the XIth International Congress of Philosophy,* Brussels, 1953, Volume 14 (Amsterdam: North-Holland Publishing Co.).



(1) Nec '$9 > 5$',

(2) Nec (Sturm's theorem),

(3) Nec 'Napoleon escaped from Elba',

in each case attaching the predicate 'Nec' to a noun, a singular term, which is a *name of* the statement which is affirmed to be necessary (or necessarily true). Of the above examples, (1) and (2) would presumably be regarded as true and (3) as false; for the necessity concerned in modal logic is generally conceived to be of a logical or a priori sort.

A second and more drastic degree in which the notion of necessity may be adopted is in the form of a *statement operator.* Here we have no longer a predicate, attaching to names of statements as in (1)–(3), but a logical operator 'nec', which attaches to statements themselves, in the manner of the negation sign. Under this usage, (1) and (3) would be rendered rather as:

(4) nec $(9 > 5)$,

(5) nec (Napoleon escaped from Elba),

and (2) would be rendered by prefixing 'nec' to Sturm's actual theorem rather than to its name. Thus whereas 'Nec' is a predicate or verb, 'is necessary', which attaches to a noun to form a statement, 'nec' is rather an adverb, 'necessarily', which attaches to a statement to form a statement.

Finally the third and gravest degree is expression of necessity by a sentence operator. This is an extension of the second degree, and goes beyond it in allowing the attachment of 'nec' not only to statements but also to open sentences, such as '$x > 5$', preparatory to the ultimate attachment of quantifiers:

(6) $(x)$ nec $(x > 5)$,

(7) $(\exists x)$ nec $(x > 5)$,

(8) $(x)[x = 9 \,.\!\supset$ nec $(x > 5)]$.

The example (6) would doubtless be rated as false, and perhaps (7) and (8) as true.

I shall be concerned in this paper to bring out the logical and philosophical significance of these three degrees of acceptance of a necessity device.

## I

I call an occurrence of a singular term in a statement *purely referential*[1] (Frege: *gerade*[2]), if, roughly speaking, the term serves in that particular context simply to refer to its object. Occurrences within quotation are not in general referential; e.g., the statements:

(9) 'Cicero' contains six letters,

(10) '9 > 5' contains just three characters

say nothing about the statesman Cicero or the number 9. Frege's criterion for referential occurrence is substitutivity of identity. Since

(11) Tully = Cicero,

(12) the number of planets = 9,

whatever is true of Cicero is true *ipso facto* of Tully (these being one and the same) and whatever is true of 9 is true of the number of planets. If by putting 'Tully' for 'Cicero' or 'the number of planets' for '9' in a truth, e.g., (9) or (10), we come out with a falsehood:

(13) 'Tully' contains six letters,

(14) 'the number of planets > 5' contains just three characters,

we may be sure that the position on which the substitution was made was not purely referential.

(9) must not be confused with:

(15) Cicero has a six-letter name,

which *does* say something about the man Cicero, and—unlike (9)—remains true when the name 'Cicero' is supplanted by 'Tully'.

Taking a hint from Russell,[3] we may speak of a context as *referentially opaque* when, by putting a statement $\phi$ into that context, we can cause a purely referential occurrence in $\phi$ to be not purely referential in the whole context. E.g., the context:

'. . .' contains just three characters

[1] *From a Logical Point of View,* pp. 75f, 139ff, 145.
[2] "Über Sinn und Bedeutung."
[3] Whitehead and Russell, 2d ed., Vol. 1, Appendix C.



is referentially opaque; for, the occurrence of '9' in '9 > 5' is purely referential, but the occurence of '9' in (10) is not. Briefly, a context is referentially opaque if it can render a referential occurrence non-referential.

Quotation is the referentially opaque context par excellence. Intuitively, what occurs inside a referentially opaque context may be looked upon as an orthographic accident, without logical status, like the occurrence of 'cat' in 'cattle'. The quotational context ''9 > 5'' of the statement '9 > 5' has, perhaps, unlike the context 'cattle' of 'cat', a deceptively systematic air which tempts us to think of its parts as somehow logically germane. Insofar as this temptation exists, it is salutary to paraphrase quotations by the following expedient. We may adopt names for each of our letters and other characters, and Tarski's '⁀' to express concatenation. Then, instead of naming a notational form by putting that notational form itself bodily between quotation marks, we can name it by spelling it. E.g., since 'μ' is mu, 'ε' is epsilon, and 'ν' is nu, the word 'μεν' is mu⁀epsilon⁀nu. Similarly the statement '9 > 5' is n⁀g⁀f, if we adopt the letters 'n', 'g', and 'f' as names of the characters '9', '>', and '5'. The example (10) can thus be transcribed as:

(16) n⁀g⁀f contains just three characters.

Here there is no non-referential occurrence of the numeral '9', for there is no occurrence of it all; and here there is no referentially opaque containment of one statement by another, because there is no contained statement at all. Paraphrasing (10) into (16), so as to get rid altogether of the opaquely contained statement '9 > 5', is like paraphrasing 'cattle' into 'kine' so as to rid it of the merely orthographic occurrence of the term 'cat'. Neither paraphrase is mandatory, but both are helpful when the irreferential occurrences draw undue attention.

An occurrence of a statement as a part of a longer statement is called *truth-functional* if, whenever we supplant the contained statement by another statement having the same truth value, the containing statement remains unchanged in truth value. Naturally one would not expect occurrences of statements within referentially opaque contexts, such as quotations, to be truth-functional. E.g., the truth (10) becomes false when the contained statement '9 > 5' is supplanted by another, 'Napoleon escaped from Elba', which has the same truth value as '$9 > 5$'. Again the truth (1) is carried, by that same substitution, into the falsehood (3). One might not expect occurrences of statements within statements to be truth-functional, in general, even when the contexts are not referentially opaque; certainly not when the contexts are referentially opaque.

In mathematical logic, however, a policy of *extensionality* is widely espoused: a policy of admitting statements within statements truth-functionally only (apart of course from such contexts as quotation, which are referentially opaque). Note that the semantical predicate 'Nec' as of (1)–(3) is reconcilable with this policy of extensionality, since whatever breach of extensionality it *prima facie* involves is shared by examples like (10) and attributable to the referential opacity of quotation. We can always switch to the spelling expedient, thus rewriting (1) as:

(17) Nec (n⁀g⁀f).

(17), like (16) and indeed (2) and unlike (1) and (3), contains no component statement but only a name of a statement.

The statement operator 'nec', on the other hand, is a premeditated departure from extensionality. The occurrence of the truth '$9 > 5$' in (4) is non-truth-functional, since by supplanting it by a different truth we can turn the true context (4) into a falsehood such as (5). Such occurrences, moreover, are not looked upon as somehow spurious or irrelevant to logical structure, like occurrences in quotation or like 'cat' in 'cattle'. On the contrary, the modal logic typified in (4) is usually put forward as a corrective of extensionality, a needed supplementation of an otherwise impoverished logic. Truth-functional occurrence is by no means the rule in ordinary language, as witness occurrences of statements governed by 'because', 'thinks that', 'wishes that', etc., as well as 'necessarily'. Modal logicians, adopting 'nec', have seen no reason to suppose that an adequate logic might adhere to a policy of extensionality.

But, for all the willingness of modal logicians to flout the policy of extensionality, is there really any difference—on the score of extensionality—between their statement operator 'nec' and the extensionally quite admissible semantical predicate 'Nec'? The latter was excusable, within a policy of extensionality, by citing the referential opacity of quotation. But the statement operator 'nec' is likewise excusable, within a policy of extensionality, by citing the referential opacity of 'nec' itself! To see the referential opacity of 'nec' we have only to note that (4) and (12) are true and yet this is false:

(18) nec (the number of planets $> 5$).



The statement operator 'nec' is, in short, on a par with quotation. (1) happens to be written with quotation marks and (4) without, but from the point of view of a policy of extensionality one is no worse than the other. (1) might be preferable to (4) only on the score of a possible ancillary policy of trying to reduce referentially opaque contexts to uniformly quotational form.

Genuine violation of the extensionality policy, by admitting non-truth-functional occurrences of statements within statements *without* referential opacity, is less easy than one at first supposes. Extensionality does not merely recommend itself on the score of simplicity and convenience; it rests on somewhat more compelling grounds, as the following argument will reveal. Think of '$p$' as short for some statement, and think of '$F(p)$' as short for some containing true statement, such that the context represented by '$F$' is not referentially opaque. Suppose further that the context represented by '$F$' is such that logical equivalents are interchangeable, within it, *salvâ veritate*. (This is true in particular of 'nec'.) What I shall show is that the occurrence of '$p$' in '$F(p)$' is then truth-functional. I.e., think of '$q$' as short for some statement having the same truth value as '$p$'; I shall show that '$F(q)$' is, like '$F(p)$', true.

What '$p$' represents is a statement, hence true or false (and devoid of free '$x$'). If '$p$' is true, then the conjunction '$x = \Lambda . p$' is true of one and only one object $x$, viz., the empty class $\Lambda$; whereas if '$p$' is false the conjunction '$x = \Lambda . p$' is true of no object $x$ whatever. The class $\hat{x}(x = \Lambda . p)$, therefore, is the unit class $\iota\Lambda$ or $\Lambda$ itself according as '$p$' is true or false. Moreover, the equation:

$$\hat{x}(x = \Lambda . p) = \iota\Lambda$$

is, by the above considerations, *logically* equivalent to '$p$'. Then, since '$F(p)$' is true and logical equivalents are interchangeable within it, this will be true:

(19) $$F[\hat{x}(x = \Lambda . p) = \iota\Lambda].$$

Since '$p$' and '$q$' are alike in truth value, the classes $\hat{x}(x = \Lambda . p)$ and $\hat{x}(x = \Lambda . q)$ are both $\iota\Lambda$ or both $\Lambda$; so

(20) $$\hat{x}(x = \Lambda . p) = \hat{x}(x = \Lambda . q).$$

Since the context represented by '$F$' is not referentially opaque, the occurrence of '$\hat{x}(x = \Lambda . p)$' in (19) is a purely referential occurrence and hence subject to the substitutivity of identity; so from (19) by (20) we can conclude that

$$F[\hat{x}(x = \Lambda . q) = \iota\Lambda].$$

Thence in turn, by the logical equivalence of '$\hat{x}(x = \Lambda . q) = \iota\Lambda$' to '$q$', we conclude that $F(q)$.

The above argument cannot be evaded by denying (20), as long as the notation in (20) is construed, as usual, as referring to classes. For classes, properly so-called, are one and the same if their members are the same—regardless of whether that sameness be a matter of logical proof or of historical accident. But the argument could be contested by one who does not admit class names '$\hat{x}( \ . \ . \ . \ )$'. It could also be contested by one who, though admitting such class names, does not see a final criterion of referential occurrence in the substitutivity of identity, as applied to constant singular terms. These points will come up, perforce, when we turn to 'nec' as a sentence operator under quantification. Meanwhile the above argument does serve to show that the policy of extensionality has more behind it than its obvious simplicity and convenience, and that any real departure from the policy (at least where logical equivalents remain interchangeable) must involve revisions of the logic of singular terms.

The simpler earlier argument for the referential opacity of the statement operator 'nec', viz., observation of the truths (4) and (12) and the falsehood (18), could likewise be contested by one who either repudiates constant singular terms or questions the criterion of referential opacity which involves them. Short of adopting 'nec' as a full-fledged *sentence* operator, however, no such searching revisions of classical mathematical logic are required. We can keep to a classical theory of classes and singular terms, and even to a policy of extensionality. We have only to recognize, in the *statement* operator 'nec', a referentially opaque context comparable to the thoroughly legitimate and very convenient context of quotation. We can even look upon (4) and (5) as elliptical renderings of (1) and (3).



## II

Something very much to the purpose of the semantical predicate 'Nec' is regularly needed in the theory of proof. When, e.g., we speak of the completeness of a deductive system of quantification theory, we have in mind some concept of *validity* as norm with which to compare the class of obtainable theorems. The notion of validity in such contexts is not identifiable with truth. A true statement is not a valid statement of quantification theory unless not only it but all other statements similar to it in quantificational structure are true. Definition of such a notion of validity presents no problem, and the importance of the notion for proof theory is incontestable.

A conspicuous derivative of the notion of quantificational validity is that of quantificational implication. One statement quantificationally implies another if the material conditional composed of the two statements is valid for quantification theory.

This reference to quantification theory is only illustrative. There are parallels for truth-function theory: a statement is valid for truth-function theory if it and all statements like it in truth-functional structure are true, and one statement truth-functionally implies another if the material conditional formed of the two statements is valid for truth-function theory.

And there are parallels, again, for logic taken as a whole: a statement is logically valid if it and all statements like it in logical structure are true, and one statement logically implies another if the material conditional formed of the two statements is logically valid.

Modal logic received special impetus years ago from a confused reading of '$\supset$', the material 'if–then', as 'implies': a confusion of the material conditional with the relation of implication.[4] Properly, whereas '$\supset$' or 'if–then' connects statements, 'implies' is a verb which connects names *of* statements and thus expresses a relation of the named statements. Carelessness over the distinction of use and mention having allowed this intrusion of 'implies' as a reading of '$\supset$', the protest thereupon arose that '$\supset$' in its material sense was too weak to do justice to 'implies', which connotes some-

[4] Notably in Whitehead and Russell.

thing like logical implication. Accordingly an effort was made to repair the discrepancy by introducing an improved substitute for '$\supset$', written '$\prec$' and called strict implication.[5] The initial failure to distinguish use from mention persisted; so '$\prec$', though read 'implies' and motivated by the connotations of the word 'implies', functioned actually not as a verb but as a statement connective, a much strengthened 'if–then'. Finally, in recognition of the fact that logical implication is validity of the material conditional, a validity operator 'nec' was adopted to implement the definition of '$p \prec q$' as 'nec $(p \supset q)$'. Since '$\prec$' had been left at the level of a statement connective, 'nec' in turn was of course rendered as an operator directly attachable to statements—whereas 'is valid', properly, is a verb attachable to a name of a statement and expressing an attribute of the statement named.[6]

In any event, the use of 'nec' as statement operator is easily converted into use of 'Nec' as semantical predicate. We have merely to supply quotation marks, thus rewriting (4) and (5) as (1) and (3). The strong 'if–then', '$\prec$', can correspondingly be rectified to a relation of implication properly so-called. What had been:

(21) the witness lied $\prec$. the witness lied $\vee$ the owner is liable,

explained as:

(22) nec (the witness lied $\supset$. the witness lied $\vee$ the owner is liable),

becomes:

(23) 'the witness lied' implies 'the witness lied $\vee$ the owner is liable',

explained as:

(24) Nec 'the witness lied $\supset$. the witness lied $\vee$ the owner is liable'.

Typically, in modal logic, laws are expressed with help of schematic letters '$p$', '$q$', etc., thus:

(25) $p \prec . p \vee q$,

(26) nec $(p \supset . p \vee q)$.

[5] Lewis, *A Survey of Symbolic Logic*, Chap. 5.

[6] On the concerns of this paragraph and the next, see also §69 of Carnap, *Logical Syntax*, and §5 of my *Mathematical Logic*.



The schematic letters are to be thought of as supplanted by any specific statements so as to yield actual cases like (21) and (22). Now just as (21) and (22) are translatable into (23) and (24), so the schemata (25) and (26) themselves might be supposed translatable as:

(27) '$p$' implies '$p \vee q$',

(28) Nec '$p \supset . p \vee q$'.

Here, however, we must beware of a subtle confusion. A quotation names precisely the expression inside it; a quoted '$p$' names the sixteenth letter of the alphabet and nothing else. Thus whereas (25) and (26) are schemata or diagrams which depict the forms of actual statements, such as (21) and (22), on the other hand (27) and (28) are *not* schemata depicting the forms of actual statements such as (23) and (24). On the contrary, (27) and (28) are not schemata at all, but actual statements: statements *about* the specific schemata '$p$', '$p \vee q$', and '$p \supset . p \vee q$' (with just those letters). Moreover, the predicates 'implies' and 'Nec' have thus far been looked upon as true only of statements, not of schemata; so in (27) and (28) they are misapplied (pending some deliberate extension of usage).

The letters '$p$' and '$q$' in (25) and (26) stand in place of statements. For translation of (25) and (26) into semantical form, on the other hand, we need some special variables which refer *to* statements and thus stand in place of names of statements. Let us use '$\phi$', '$\psi$', etc., for that purpose. Then the analogues of (25) and (26) in semantical form can be rendered:

(29) $\phi$ implies the alternation of $\phi$ and $\psi$,

(30) Nec (the conditional of $\phi$ with the alternation of $\phi$ and $\psi$).

We can condense (29) and (30) by use of a conventional notation which I have elsewhere[7] called *quasi-quotation*, thus:

(31) $\phi$ implies $\ulcorner \phi \vee \psi \urcorner$,

(32) Nec $\ulcorner \phi \supset . \phi \vee \psi \urcorner$.

The relationship between the modal logic of statement operators and the semantical approach, which was pretty simple and obvious when we compared (21)–(22) with (23)–(24), is thus seen to take on some slight measure of subtlety at the stage of

[7] *Mathematical Logic*, §6.

(25)–(26); these correspond not to (27)–(28) but to (31)–(32). It is schemata like (25)–(26), moreover, and not actual statements like (21)–(22), that fill the pages of works on modal logic. However, be that as it may, it is in actual statements such as (21)–(24) that the point of modal logic lies, and it is the comparison of (21)–(22) with (23)–(24) that reflects the true relationship between the use of statement operators and that of semantical predicates. Schemata such as (25)–(26) are mere heuristic devices, useful in expounding the theory of (21)–(22) and their like; and the heuristic devices which bear similarly on (23)–(24) are (31)–(32).

Seeing how modal statement operators can be converted into semantical predicates, one may of course just note the conversion as a principle and leave it undone in practice. But there are five reasons why it is important to note it in principle. One is that the inclination to condemn '$\supset$' unduly, through a wrong association of 'if–then' with 'implies', is thereby removed. A second reason is that it is at the semantical or proof-theoretic level, where we talk *about* expressions and their truth values under various substitutions, that we make clear and useful sense of logical validity; and it is logical validity that comes nearest to being a clear explication of 'Nec', taken as a semantical predicate. A third reason is that in using 'Nec' as a semantical predicate we flaunt a familiar reminder of referential opacity, in the form of quotation marks. A fourth reason is that the adoption of 'nec' as a statement operator tempts one to go a step further and use it as a sentence operator subject to quantification. The momentousness of this further step—whereof more anon—tends to be overlooked save as one expressly conceives of the 'nec', in its use as statement operator, as shorthand for the semantical usage.

A fifth reason has to do with iteration. Since 'nec' attaches to a statement and produces a statement, 'nec' can then be applied again. On the other hand 'Nec' attaches to a name and yields a statement, to which, therefore, it cannot be applied again. An iterated 'nec', e.g.:

(33) nec nec$(x)(x$ is red $\supset x$ is red),

can of course be translated by our regular procedure into semantical form thus:

(34) Nec 'Nec '$(x)(x$ is red $\supset x$ is red)' ',



and we are thereby reminded that 'Nec' can indeed be iterated if we insert new quotation marks as needed. But the fact remains that (34) is, in contrast with (33), an unlikely move. For, suppose we have made fair sense of 'Nec' as logical validity, relative say to the logic of truth functions, quantification, and perhaps classes. The statement:

(35) $(x)(x$ is red $\supset x$ is red),

then, is typical of the statements to which we would attribute such validity; so

(36) Nec '$(x)(x$ is red $\supset x$ is red)'.

The validity of (35) resides in the fact that (35) is true and so are all other statements with the same quantificational and truth-functional structure as (35). Thus it is that (36) is *true*. But if (36) in turn is also *valid*, it is valid only in an extended sense with which we are not likely to have been previously concerned: a sense involving not only quantificational and truth-functional structure but also the semantical structure, somehow, of quotation and 'Nec' itself.

Ordinarily we work in a metalanguage, as in (36), treating of an object language, exemplified by (35). We would not rise to (34) except in the rare case where we want to treat the metalanguage by means of itself, and want furthermore to extend the notion of validity beyond the semantics of logic to the semantics of semantics. When on the other hand the statement operator 'nec' is used, iteration as in (33) is the most natural of steps; and it is significant that in modal logic there has been some question as to just what might most suitably be postulated regarding such iteration.[8]

The iterations need not of course be consecutive. In the use of modal statement operators we are led also into complex iterations such as:

(37) $p \prec q \,.\!\prec\!.\, \sim q \prec \sim p,$

short for:

(38) nec [nec $(p \supset q) \supset$ nec $(\sim q \supset \sim p)$].

[8] Cf. Lewis and Langford, pp. 497ff.

Or, to take an actual example:

(39) $(x)(x \text{ has mass}) \prec (\exists x)(x \text{ has mass}) . \prec . \sim (\exists x)(x \text{ has mass}) \prec \sim (x)(x \text{ has mass})$,

(40) $\text{nec } \{\text{nec } [(x)(x \text{ has mass}) \supset (\exists x)(x \text{ has mass})] \supset \text{nec } [\sim(\exists x)(x \text{ has mass}) \supset \sim (x)(x \text{ has mass})]\}$ .

In terms of semantical predicates the correspondents of (39) and (40) are:

(41) ' '$(x)(x$ has mass$)$' implies '$(\exists x)(x$ has mass$)$' ' implies ' '$\sim(\exists x)(x$ has mass$)$' implies '$\sim(x)(x$ has mass$)$' ',

(42) Nec 'Nec '$(x)(x$ has mass$) \supset (\exists x)(x$ has mass$)$' $\supset$ Nec '$\sim(\exists x)(x$ has mass$) \supset \sim (x)(x$ has mass$)$' '.

But (41)–(42), like (34), have singularly little interest or motivation when we think of necessity semantically.

It is important to note that we must not translate the schemata (37)–(38) into semantical form in the manner:

' '$p$' implies '$q$' ' implies, etc.

To do so would be to compound, to an altogether horrifying degree, the error noted earlier of equating (25)–(26) to (27)–(28). The analogues of (37)–(38) in semantical application should be rendered rather:

(43) $\ulcorner\phi$ implies $\psi\urcorner$ implies $\ulcorner\ulcorner\sim\psi\urcorner$ implies $\ulcorner\sim\phi\urcorner\urcorner$,

(44) Nec $\ulcorner$Nec $\ulcorner\phi \supset \psi\urcorner \supset$ Nec $\ulcorner\sim\psi \supset \sim\phi\urcorner\urcorner$,

subject to some special conventions governing the nesting of quasi-quotations. Such conventions would turn on certain subtle considerations which will not be entered upon here. Suffice it to recall that the sort of thing formulated in (33)–(34) and (37)–(44) is precisely the sort of thing we are likely to see least point in formulating when we think of necessity strictly as a semantical predicate rather than a statement operator. It is impressive and significant that *most* of modal logic (short of quantified modal logic, to which we shall soon turn) is taken up with iterated cases like (33) and (37)–(40) which would simply not recommend themselves to our attention if necessity were held to the status of a semantical predicate and not depressed to the level of a statement operator.

Our reflections have favored the semantical side immensely,



but they must not be allowed to obscure the fact that even as a semantical predicate necessity can raise grave questions. There is no difficulty as long as necessity is construed as validity relative say to the logic of truth functions and quantification and perhaps classes. If we think of arithmetic as reduced to class theory, then such validity covers also the truths of arithmetic. But one tends to include further territory still; cases such as 'No bachelor is married', whose truth is supposed to depend on "meanings of terms" or on "synonymy" (e.g., the synonymy of 'bachelor' and 'man not married'). The synonymy relation on which such cases depend is supposedly a narrower relation than that of the mere coextensiveness of terms, and it is not known to be amenable to any satisfactory analysis. In short, necessity in semantical application tends to be identified with what philosophers call analyticity; and analyticity, I have argued elsewhere,[9] is a pseudo-concept which philosophy would be better off without.

As long as necessity in semantical application is construed simply as explicit truth-functional validity, on the other hand, or quantificational validity, or set-theoretic validity, or validity of any other well-determined kind, the logic of the semantical necessity predicate is a significant and very central strand of proof theory. But it is not modal logic, even unquantified modal logic, as the latter ordinarily presents itself; for it is a remarkably meager thing, bereft of all the complexities which are encouraged by the use of 'nec' as a statement operator. It is unquantified modal logic minus all principles which, explicitly or implicitly (via '$\prec$', etc.), involve iteration of necessity; and plus, if we are literal-minded, a pair of quotation marks after each 'Nec'.

## III

Having adopted the operator '$\sim$' of negation as applicable to statements, one applies it without second thought to open sentences as well: sentences containing free variables ripe for quantification. Thus we can write not only '$\sim$(Socrates is mortal)' but also '$\sim(x$ is mortal$)$', from which, by quantification

[9] "Two dogmas of empiricism."

and further negation, we have '$\sim(x) \sim(x$ is mortal)' or briefly '$(\exists x)$ ($x$ is mortal)'. With negation this is as it should be. As long as 'nec' is used as a statement operator, on a par with negation, the analogous course suggests itself again: we write not only 'nec $(9 > 5)$' but also 'nec $(x > 5)$', from which by quantification we can form (6)–(8) and the like.

This step brings us to 'nec' as sentence operator, Given 'nec' as statement operator, the step is natural. Yet it is a drastic one, for it suddenly obstructs the earlier expedient of translation into terms of 'Nec' as semantical predicate. We can reconstrue (4) and (5) at will as (1) and (3), but we cannot reconstrue:

(45) nec $(x > 5)$

correspondingly as:

(46) Nec '$x > 5$'.

'Nec' has been understood up to now as a predicate true only of statements, whereas (46) attributes it rather to an open sentence and is thus trivially false, at least pending some deliberate extension of usage. More important, whereas (45) is an open sentence with free '$x$', (46) has no corresponding generality; (46) is simply a statement *about* a specific open sentence. For, it must be remembered that '$x > 5$' in quotation marks is a name of the specific quoted expression, with fixed letter '$x$'. The '$x$' in (46) cannot be reached by a quantifier. To write:

(47) $(x)$(Nec '$x > 5$'), $(\exists x)$(Nec '$x > 5$')

is like writing:

(48) $(x)$(Socrates is mortal), $(\exists x)$(Socrates is mortal);

the quantifier is followed by no germane occurrence of its variable. In a word, necessity as sentence operator does not go over into terms of necessity as semantical predicate.

Moreover, acceptance of necessity as a sentence operator implies an attitude quite opposite to our earlier one (in §§I–II above), which was that 'nec' as statement operator is referentially opaque. For, one would clearly have no business quantifying into a referentially opaque context; witness (47) above. We can reasonably infer '$(\exists x)$ nec $(x > 5)$' from 'nec $(9 > 5)$' only if we regard the latter as telling us something about the *object* 9, a number, viz. that it necessarily exceeds 5. If 'nec $( \ldots > 5)$' can turn out true or false "of" the number 9 depending merely on how that number is referred to (as the falsity of (18) suggests), then evidently 'nec $(x > 5)$' expresses no genuine condition on objects of any kind. If the occurrence of '9' in 'nec $(9 > 5)$' is not purely referential, then putting '$x$' for '9' in 'nec $(9 > 5)$' makes no more sense than putting '$x$' for 'nine' within the context 'canine'.



But isn't it settled by the truth of (4) and (12) and the falsity of (18) that the occurrence of '9' in question is irreferential, and more generally that 'nec' is referentially opaque, and hence that 'nec' as a sentence operator under quantifiers is a mistake? No, not if one is prepared to accede to certain pretty drastic departures, as we shall see.

Thus far we have tentatively condemned necessity as general sentence operator on the ground that 'nec' is referentially opaque. Its referential opacity has been shown by a breakdown in the operation of putting one constant singular term for another which names the same object. But it may justly be protested that constant singular terms are a notational accident, not needed at the level of primitive notation.

For it is well known that primitively nothing in the way of singular terms is needed except the variables of quantification themselves. Derivatively all manner of singular terms may be introduced by contextual definition in conformity with Russell's theory of singular descriptions. Class names, in particular, which figured in the general argument for extensionality in §I above, may be got either by explaining '$\hat{x}( \ldots )$' as short for the contextually defined description '$(\imath y)(x)(x \in y . \equiv \ldots)$' or by adopting a separate set of contextual definitions for the purpose.[10]

Now the modal logician intent on quantifying into 'nec' sentences may say that 'nec' is not referentially opaque, but that it merely interferes somewhat with the contextual definition of singular terms. He may argue that '$(\exists x)$ nec $(x > 5)$' is not meaningless but true, and in particular that the number 9 is one of the things of which 'nec $(x > 5)$' is true. He may blame the real or apparent discrepancy in truth value between (4) and (18) simply on a queer behavior of contextually defined singular terms. Specifically he may hold that (18) is true if construed as:

[10]Cf. my *Methods of Logic*, §§36–38 (3d ed., §§41–43); *Mathematical Logic*, §§24, 26.

(49) $(\exists x)$[there are exactly $x$ planets . nec $(x > 5)$]

and false if construed as:

(50) nec $(\exists x)$(there are exactly $x$ planets . $x > 5$),

and that (18) as it stands is ambiguous for lack of a distinguishing mark favoring (49) or (50).[11] No such ambiguity arises in the contextual definition of a singular term in extensional logic (as long as the named object exists), and our modal logician may well deplore the complications which thus issue from the presence of 'nec' in his primitive notation. Still he can fairly protest that the erratic behavior of contextually defined singular terms is no reflection on the meaningfulness of his primitive notation, including his open 'nec' sentences and his quantification of them.

Looking upon quantification as fundamental, and constant singular terms as contextually defined, one must indeed concede the inconclusiveness of a criterion of referential opacity that rests on interchanges of constant singular terms. The objects of a theory are not properly describable as the things named by the singular terms; they are the values, rather, of the variables of quantification.[12] Fundamentally the proper criterion of referential opacity turns on quantification rather than naming, and is this: a referentially opaque context is one that cannot properly be *quantified into* (with quantifier outside the context and variable inside). Quotation, again, is the referentially opaque context par excellence; cf. (47). However, to object to necessity as sentence operator on the grounds of referential opacity so defined would be simply to beg the question.

Frege's criterion of referential occurrence, viz., substitutivity of identity, underlay the notion of referential opacity as developed in §I above. The statements of identity there concerned were formed of constant singular terms; cf. (11), (12). But there is a more fundamental form of the law of substitutivity of identity, which involves no constant singular terms, but only variables of quantification; viz.:

(51) $(x)(y)(x = y \,.\supset.\, Fx \equiv Fy)$.

This law is independent of any theory of singular terms, and cannot properly be challenged. For, to challenge it were simply to

[11] Thus Smullyan.
[12] See *From a Logical Point of View*, pp. 12ff, 75f, 102–110, 113ff, 148ff.



use the sign '=' in some unaccustomed way irrelevant to our inquiry. In any theory, whatever the shapes of its symbols, an open sentence whose free variables are '$x$' and '$y$' is an expression of identity only in case it fulfills (51) in the role of '$x = y$'. The generality of '$F$' in (51) is this: '$Fx$' is to be interpretable as any open sentence of the system in question, having '$x$' as free (quantifiable) variable; and '$Fy$', of course, is to be a corresponding context of '$y$'.

If 'nec' is not referentially opaque, '$Fx$' and '$Fy$' in (51) can in particular be taken respectively as 'nec $(x = x)$' and 'nec $(x = y)$'. From (51), therefore, since surely 'nec $(x = x)$' is true for all $x$, we have:

(52) $(x)(y)[x = y \,.\supset$ nec $(x = y)]$.

I.e., identity holds necessarily if it holds at all.

Let us not jump to the conclusion, just because (12) is true, that

(53) nec (the number of planets = 9).

This does not follow from (12) and (52) except with help of a law of universal instantiation, allowing us to put singular terms 'the number of planets' and '9' for the universally quantified '$x$' and '$y$' of (52). Such instantiation is allowable, certainly, in extensional logic; but it is a question of good behavior of constant singular terms, and we have lately observed that such behavior is not to be counted on when there is a 'nec' in the woodpile.

So our observations on necessity in quantificational application are, up to now, as follows. Necessity in such application is not *prima facie* absurd if we accept some interference in the contextual definition of singular terms. The effect of this interference is that constant singular terms cannot be manipulated with the customary freedom, even when their objects exist. In particular they cannot be used to instantiate universal quantifications, unless special supporting lemmas are at hand. A further effect of necessity in quantificational application is that objects come to be necessarily identical if identical at all.

There is yet a further consequence, and a particularly striking one: Aristotelian essentialism. This is the doctrine that some of the attributes of a thing (quite independently of the language in which the thing is referred to, if at all) may be essential to the thing, and others accidental. E.g., a man, or talking animal, or featherless biped (for they are in fact all the same *things*), is essentially rational and accidentally two-legged and talkative, not merely qua man but qua itself. More formally, what Aristotelian essentialism says is that you can have open sentences—which I shall represent here as '$Fx$' and '$Gx$'—such that

(54) $$(\exists x)(\text{nec } Fx \,.\, Gx \,.\, \sim \text{nec } Gx).$$

An example of (54) related to the falsity of (53) might be:

$$(\exists x)[\text{nec}(x > 5) \,.\, \text{there are just } x \text{ planets} \,.\, \sim\text{nec (there are just } x \text{ planets)}],$$

such an object $x$ being the number (by whatever name) which is variously known as 9 and the number of planets.

How Aristotelian essentialism as above formulated is required by quantified modal logic can be quickly shown. Actually something yet stronger can be shown: that there are open sentences '$Fx$' and '$Gx$' fulfilling not merely (54) but:

$$(x)(\text{nec } Fx \,.\, Gx \,.\, \sim \text{nec } Gx),$$

i.e.:

$$(x) \text{ nec } Fx \,.\, (x)\, Gx \,.\, (x) \sim \text{nec } Gx.$$

An appropriate choice of '$Fx$' is easy: '$x = x$'. And an appropriate choice of '$Gx$' is '$x = x \,.\, p$', where in place of '$p$' any statement is chosen which is true but not necessarily true. Surely there *is* such a statement, for otherwise 'nec' would be a vacuous operator and there would be no point in modal logic.

Necessity as semantical predicate reflects a non-Aristotelian view of necessity: necessity resides in the way in which we say things, and not in the things we talk about. Necessity as statement operator is capable, we saw, of being reconstrued in terms of necessity as a semantical predicate, but has, nevertheless, its special dangers; it makes for an excessive and idle elaboration of laws of iterated modality, and it tempts one to a final plunge into quantified modality. This last complicates the logic of singular terms; worse, it leads us back into the metaphysical jungle of Aristotelian essentialism.